Uplift histories from river profiles

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[1] Longitudinal river profiles, where elevation of a river bed is plotted as a function of distance along the river bed, contain information about uplift rate. When a region adjacent to a reference level (e.g., sea level) is uplifted, a rapid change in gradient occurs near the river mouth. The erosional process causes this change in gradient to migrate upstream. Thus a river profile is effectively a 'tape recording' of the uplift rate history, provided that the erosional process can be adequately parameterized. Here, we use a non-linear equation to relate the shape of a river profile, $z(x)$, to uplift rate history, $U(t)$. If erosion is assumed to be dominated by knickpoint retreat, an inverse model can be formulated and used to calculate uplift rate histories. Our model builds upon standard stream profile analysis, which focuses on the relationship between profile slope and drainage area. We have applied this analytical approach to river profiles from the Bié Dome, Angola. Calculated uplift rate histories agree with independent geologic estimates.
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1. Introduction

[2] Despite their importance, reliable estimates of surface uplift on tectonically significant timescales and length scales (i.e., 1–100 Myrs, 10–1000 km) are difficult to obtain. Here, we investigate how longitudinal river profiles can be used to calculate temporal changes in surface uplift. This topic is of considerable interest to geomorphologists and there is a long history of investigation which goes back to the pioneering work of *Davis* [1899].

[3] Our principal aim is to develop and apply an analytical inverse model which extracts uplift rate history from an observed river profile. The approach we use builds upon a large number of previous contributions [e.g., *Whipple and Tucker*, 1999; *Royden et al.*, 2000; *Whipple and Tucker*, 2002; *Clark et al.*, 2006; *Pelletier*, 2007; *Bishop*, 2007]. We assume that elevation along a river profile is controlled by the history of uplift rate and moderated by the erosional process. For simplicity, we also assume that drainage planforms do not vary and that sea level (i.e., base level) is fixed.

2. Theory

2.1. Mathematical Model

[4] It is accepted that river profiles evolve according to a governing equation that relates changes in elevation, $z(x, t)$, to the uplift rate, U, and to erosion rate, E. Thus

$$\frac{\partial z}{\partial t} = U(x,t) - E(x,t),$$

where x is the downstream distance from the river source and t is time. The general problem, in which U varies as a function of time and space, is important but it is less amenable to an analytical approach than the problem in which U is independent of x. This simplified problem is relevant for two reasons. First, U is particularly significant at sea level where there is an enforced boundary condition $z = 0$. Secondly, the spatial variation of U is often reasonably smooth [*Al-Hajri et al.*, 2009].

[5] $E(x, t)$ parameterizes a complex set of processes, which is often approximated using two basic terms. The first term is advective and concerns stream power, which is often used as a proxy for rate of incision since it is primarily controlled by water discharge [*Whipple and Tucker*, 1999]. The extent to which rate of incision is also affected by sedimentary flux, lithology and channel width is much debated [*Sklar and Dietrich*, 2001; *Amos and Burbank*, 2007]. For simplicity, we assume that discharge varies according to some power of x [*Hack*, 1957; *Weissel and Seidl*, 1998]. A second term determines overall lowering of a river profile, which can be modeled as a diffusive process varying along the profile. Thus our governing law takes the form

$$\frac{\partial z}{\partial t} = U(t) - v_\circ x^m \left(-\frac{\partial z}{\partial x}\right)^n + \kappa(x)\frac{\partial^2 z}{\partial x^2}, \qquad (1)$$

where $v_\circ$ is the knickpoint velocity if $m = 0$ and $n = 1$, m and n are dimensionless parameters, and κ is the diffusivity. The x^m term represents discharge, which increases downstream. Note that *Whipple and Tucker* [1999] and others use $A^{\hat{m}}$ as a proxy for discharge, where A is the upstream drainage area at any position x. If the drainage planform has an aspect ratio of $\sim$1, then $\hat{m} \sim 0.5m$. A natural boundary condition for equation (1) is $z(L, t) = 0$ where L is river length. If river length changes as a function of time are small compared with L, this boundary condition is reasonable. Alternative boundary conditions are considered by *Smith et al.* [2000].

[6] The relative importance of advective (i.e., detachment-limited) and diffusive (i.e., transport-limited) terms is determined by a morphodynamic Péclet number

$$Pe = \kappa^{-1} v_\circ^{1/n} L^{1+m/n} U_\circ^{1-1/n},$$

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where $U_\circ$ is a typical uplift rate. When $Pe \gg 1$, the diffusive term is negligible and equation (1) reduces to

$$\frac{\partial z}{\partial t} + v_\circ x^m \left(-\frac{\partial z}{\partial x}\right)^n = U(t). \tag{2}$$

[7] Upstream portions of many river profiles are often just as rough as downstream portions (i.e., both portions have similar frequency contents), which suggests that neglecting the diffusive term can be justified. Equation (2) is a kinematic wave equation in which a signal propagating upstream is encoded in the gradient of the profile [*Whitham*, 1974]. The boundary condition $z(L, t) = 0$ immediately relates the gradient to the uplift rate so that

$$\left.\frac{\partial z}{\partial x}\right|_{x=L} = -\left[\frac{U(t)}{v_\circ L^m}\right]^{1/n}. \tag{3}$$

2.2. Inverse Model

2.2.1. Model Simplification

[8] The way in which an uplift signal propagates upstream is more apparent if equation (2) is differentiated with respect to x and then rewritten in terms of a rescaled distance co-ordinate, ξ, and a rescaled gradient variable, θ, which are defined by

$$\xi = \left(\frac{x}{L}\right)^{1-m/n}, \qquad \theta = -v_\circ^{1/n} x^{m/n} \frac{\partial z}{\partial x}.$$

This substitution [*Smith et al.*, 2000] yields

$$\frac{\partial \theta}{\partial t} = V_\circ \theta^{n-1} \frac{\partial \theta}{\partial \xi}, \quad \text{where} \quad V_\circ = (n-m) v_\circ^{1/n} L^{m/n-1}. \tag{4}$$

[9] Thus, a signal, expressed as a constant value of θ, propagates upstream along a characteristic path defined by

$$\frac{d\xi}{dt} = -V_\circ \theta^{n-1}.$$

[10] We know that each path originates at $\xi = 1$ at a time $t_\circ$. According to the boundary condition given by equation (3), the value of θ that it carries is given by

$$\theta = -v_\circ^{1/n} L^{m/n} \left.\frac{\partial z}{\partial x}\right|_{(L,t_\circ)} = [U(t_\circ)]^{1/n}.$$

[11] This representation makes it straightforward to reconstruct the uplift rate history. If the river profile, $\theta(x)$, is known at a point x and at a time t (i.e. the present day), the signal must have originated from the mouth at a time $\tau = t - (1 - (x/L)^{1-m/n})/(V_\circ[\theta(x)]^{n-1})$. Uplift rate at that time is given by $U(\tau) = [\theta(x)]^n$. Given profile information $z(x)$ at $t = 0$, then

$$U(\tau) = v_\circ x^m \left(-\frac{\partial z}{\partial x}\right)^n \tag{5}$$

where

$$\tau = -\frac{L^{1-m}}{(n-m)v_\circ}\left[\frac{1-(x/L)^{1-m/n}}{(x/L)^{m(n-1)/n}}\right]\left(-\frac{\partial z}{\partial x}\right)^{1-n}. \tag{6}$$

[12] In a limited sense, our analytical transient solutions are implicit in slope-drainage area analysis [e.g., *Clark et al.*, 2006]. It is widely recognized that uplifted profile segments propagate upstream at a given rate. This information about channel steepness and knickpoint position could be used to infer a partial uplift history [*Niemann et al.*, 2001; *Wobus et al.*, 2006a, 2006b; *Weissel and Seidl*, 1998]. Equations (5) and (6) enable complete uplift histories to be deduced in an explicit and quantitative fashion.

2.2.2. Shocks, Gaps and Uniqueness

[13] Solutions to equation (4) develop shocks if $n > 1$. Shocks occur when a steeper reach of the river, which is propagating rapidly upstream, overtakes a less steep reach, which is propagating more slowly upstream. From the kinematic-wave solution, a shock forms within the domain $0 \leq x \leq L$ if, and only if,

$$\left(\frac{n-1}{n}\right)\frac{dU}{dt} > (n-m) v_\circ^{1/n} L^{m/n-1} [U(t)]^{2-1/n} \tag{7}$$

at some time during uplift. If so, the river erases part of its uplift rate history and the reconstructed record will contain a gap. Although this condition only strictly holds for the detachment-limited model considered here, it implies that if uplift histories were reconstructed using the more general model described by equation (1), sharply convex-upwards regions of the profile would 'smear out' into substantial tracts of reconstructed history. Thus in any model, periods during which equation (7) holds, or is approached, may be sensitive to artefacts or to fine details of a river profile.

[14] Inverse modeling may also fail if it predicts a time history, $\tau(x)$, which does not depend monotonically on x, so that a reach of the river appears to be older than the region immediately upstream of it. If this situation arises, it implies that information is not solely propagating upstream from the river mouth. A possible explanation is that $U(x, t)$ is not spatially uniform and so gradient information can be 'fed into' the system at all points, not just at $x = L$. Alternatively, other processes not represented by equation (2) have come into play. $\partial\tau/\partial x$ will be positive if

$$\frac{\partial^2 z}{\partial x^2} < \left(-\frac{\partial z}{\partial x}\right)\frac{1}{x}\frac{\left[(1-m) + \left(m - \frac{m}{n}\right)(x/L)^{m/n-1}\right]}{\left[(x/L)^{m/n-1} - 1\right]}. \tag{8}$$

[15] Since the right-hand side of this inequality is always positive, we conclude that convex-upward profiles do not suffer from this pathology: $\tau(x)$ only ceases to be monotonic for strongly concave-upward profiles. The condition given in equation (8) provides a useful warning that assumptions which underpin the inverse model no longer hold.

2.3. Parameter Values

2.3.1. Sensitivity

[16] It is straightforward to assess the sensitivity of uplift rate histories calculated using equations (5) and (6). For

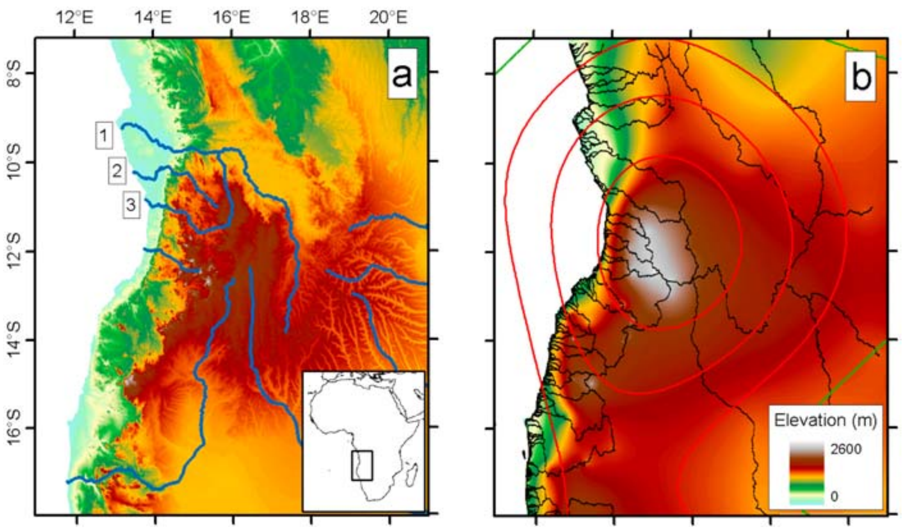

Figure 1. (a) Topography and drainage of the Bié dome, Angola (see inset for location). Blue lines = rivers; numbered lines = modeled rivers, 1 = Cuanza, 2 = Longa, 3 = Cuvo. (b) Surface fitted to network of drainage divides. Black lines = loci of drainage divides. Green contour = zero value gravity anomaly; red contours = positive gravity anomalies plotted every 10 mGal. Long wavelength (>800 km) free-air gravity anomalies were extracted from GRACE data set [*Tapley et al.*, 2005].

obvious reasons, we are particularly concerned with the way in which uplift histories are affected by changes in independently estimated parameters (e.g., $v_\circ$, m, n) and in measured values (e.g., x, L, $\partial z/\partial x$). Since $U(\tau)$ and $\tau(x)$ are proportional and inversely proportional to $v_\circ$, respectively, the main effect of varying this parameter is to rescale time: increasing $v_\circ$ by a factor α gives estimates which involve faster uplift (by a factor α) occurring over a shorter period (by a factor $1/\alpha$). Uncertainties in the value of the gradient $\partial z/\partial x$ have a larger effect on the value of $U(\tau)$, through the exponent $n \geq 1$, than on the value of τ, through the exponent $(1 - n)$. In particular, if $n = 1$, then estimates of τ are independent of gradient. The variation of τ through the prefactor L^{1-m} and of U through the factor x^m are weak, but values of x and L are important through the terms in equation (6) which involve x/L (the relative position along the reach). This factor introduces absolute errors which are more sensitive to the value of x/L in the upper reaches ($x/L \rightarrow 0$) but relative errors which are also highly sensitive in the lower reaches ($x/L \rightarrow 1$). Errors introduced by mis-estimating x/L are lowest midway along the reach, suggesting that most reliance should be placed on measurements in this range.

2.3.2. Calibration

[17] In our formulation, three erosional parameters $v_\circ$, m and n control the value of the advective term, which determines the knickpoint velocity and thus the transient form of a river profile. m determines the magnitude of x^m, which is a proxy for discharge variation along the river. m is expected to vary between 0.7 and 1.2 (i.e., $\hat{m} = 0.35-0.6$; [*Schoenbohm et al.*, 2004]). *Roberts and White* [2009] show that $v_\circ$ and m trade off against each other so that larger values of m imply smaller values of $v_\circ$ or vice versa. The value of n is widely debated (compare *Wobus et al.* [2006b] and *Pelletier* [2007]).

[18] In the absence of evidence which supports shock behavior along river profiles, we suggest that n is not significantly greater than 1. River profiles contain no information about the timescale of uplift and the three erosional parameters must be calibrated using independent geologic observations. Analysis of rivers draining Africa's topographic swells suggests that $v_\circ = 50$ m^{1-m} Myr^{-1}, $m = 0.5$ and $n = 1$. These values were chosen to yield uplift rate histories which are consistent with independent geologic estimates [see *Roberts and White*, 2009].

3. Application

3.1. Bié Dome

[19] We apply the inverse model described above to the Bié Dome, a large topographic swell which straddles the west coast of Africa (Figure 1). *Jackson et al.* [2005] and *Al-Hajri et al.* [2009] have used a variety of geologic and geophysical observations to shown that this dome grew rapidly during the last 30 Myrs with much growth occurring in the last 5 Ma. Block faulting has not played a significant role during domal growth. Along the coastal shelf, Pliocene-aged (i.e., 5–2 Ma) deltaic foreset strata are truncated at the sea bed. The center of the dome is at 12°S, 15°E and the summit envelope, which is a surface fitted to the network of drainage divide loci, attests to the smoothness of the original (i.e., uneroded) swell (Figure 1b). The relationship between the prominent free-air gravity anomaly and long wavelength topography suggest that the Bié Dome was generated by convective upwelling beneath the lithospheric plate [*Burke and Gunnell*, 2008; *Al-Hajri et al.*, 2009].

[20] Drainage of the Bié Dome is approximately radial and river profiles are consistently convex upward with dramatic knickpoints [*Lucazeau et al.*, 2003]. Longer wavelength knickpoints do not coincide with lithological changes and are more likely to reflect uplift events (Figure 2; *Roberts and White* [2009]). A digital elevation model was created using the Shuttle Radar Topographic Mission (SRTM) data set and a drainage network was extracted

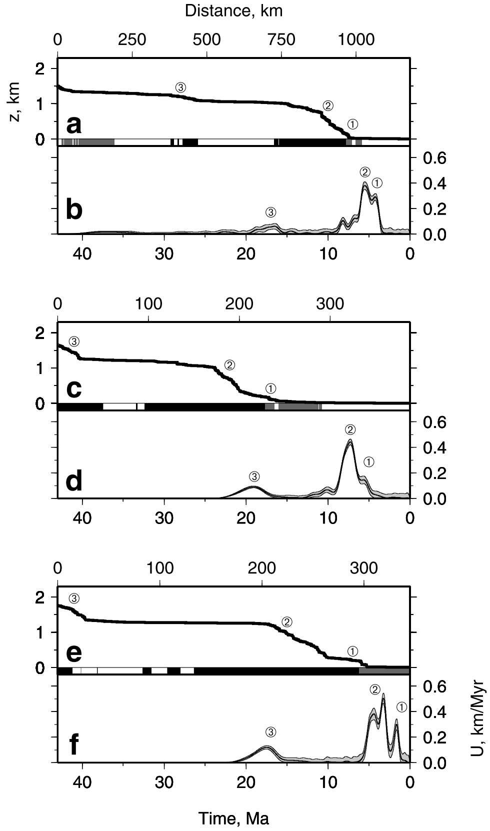

Figure 2. Inverse modelling of river profiles (see Figure 1 for location). (a) Cuanza, (c) Longa, and (e) Cuvo, longitudinal river profiles extracted from SRTM topographic data using flow-routing algorithms from ESRI ArcView software. Black, gray and white bars along base = Precambrian basement, Paleozoic/Mesozoic sedimentary rocks, and Cenozoic sedimentary rocks, respectively. Significant knickpoints do not coincide with lithologic change. (b, d and f) Calculated smooth uplift rate histories. Increases in uplift rate are numbered to aid comparison with knickpoint loci on river profiles. Gray band indicates uncertainty produced by an elevation error of ±200 m.

using a standard flow-routing algorithm. We assume that the vertical resolution of extracted profiles is a conservative ±200 m.

3.2. Reconstructing the Uplift History

[21] We have modeled three rivers which drain to the west, reaching the coast at about 14°E. The coastline at this location has been uplifted as the dome grew, creating steep gradients on the outflowing rivers. Over time, these gradients have propagated upstream. An important advantage of modeling river profiles, which are short compared with the dimensions of the topographic swell, is that U can be regarded as a function of geologic time alone. We have not modeled longer profiles, which drain away from the topographic swell since U has greater spatial variation.

[22] Figure 2 shows the $z(x)$ profiles for each river together with uplift rate histories which were calculated using equations (5) and (6). The gradient at each point along a river profile was calculated by measuring the difference between adjacent samples and applying a simple smoothing algorithm. Our results suggest that the most important phase of uplift occurred between 6 and 2 Myrs when uplift rates peaked at 0.4 km Myr^{-1}. There is good evidence for an earlier, smaller, phase of uplift. Both phases of uplift are consistent with independent geologic evidence [e.g., *Leturmy et al.*, 2003; *Lucazeau et al.*, 2003; *Jackson et al.*, 2005; *Al-Hajri et al.*, 2009]. How well constrained are these calculated uplift rate histories? There are two important sources of uncertainty. First, values of the erosional parameters may change as a function of space and time [*Whipple and Tucker*, 1999]. If n is greater than 1 but less than 1.05, the calculated $U(t)$ is gradually shifted back in time by 2–3 Myrs. If n is greater than 1.05, multiple values of $U(t)$ can occur at a given time and our underlying assumptions no longer hold. We assume that $m = 0.5$, which is smaller than the published range. If $m = 0.7$ and $v_\circ = 3.8$, the younger uplift event can be up to 1 Myrs younger. The older event can be up to 5 Myrs older. If $m = 1.2$ and $v_\circ = 0.0125$, larger displacements occur. *Roberts and White* [2009] favor $m = 0.5$ (i.e., $\hat{m} = 0.25$), which yields a set of results consistent with geologic observations of the cumulative amount of uplift [e.g., *Al-Hajri et al.*, 2009]. Secondly, profile length, L, exerts a significant influence on the calculated distribution of $U(t)$. When sea level varies, the coastline migrates across the distal end of the profile (i.e., L is not constant). If the position of the coastline varies by ±45 km, the calculated uplift rate history varies by ±1.5 Myrs.

4. Conclusions

[23] River profiles act as tectonic 'tape recorders', encoding information about uplift rate history. In general terms, reconstructing uplift rate as a function of time and space is a difficult inverse problem, which is best tackled using a numerical scheme [*Roberts and White*, 2009]. Nevertheless, it is fruitful to analyze a simplified version of this problem where uplift rate varies as a function of time only and where incision is entirely detachment-limited. Under these circumstances, the direct inverse problem can be solved. We have applied this method to river profiles from the Bié Dome, a convectively supported topographic swell which straddles the west coast of Africa. If our choice of erosional parameters is acceptable, we conclude that this dome underwent two phases of rapid Neogene uplift. These results are consistent with independent geologic observations. Our inverse algorithm can be downloaded from http://bullard.esc.cam.ac.uk/basinresearch/riverrun and used with appropriate caution.

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