Estimates of $T_e$ for Continental Regions using GOCE gravity

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Abstract

Satellite-only gravity fields and surface gravity obtained from altimetric measurements now agree well at wavelengths greater than $\sim 180$ km. Satellite gravity fields can therefore be used to estimate the elastic thickness $T_e$ in regions where surface observations are sparse. They are used for this purpose in a number of continental regions, of India, Africa, and Antarctica, where the topography is sufficiently rough, and also in regions of the USA, China, Australia and Siberia, where there are surface measurements. Estimates of $T_e$ for Antarctica depend on measurements of ice thickness, which are now available for much of the continent. Values of $T_e$ are obtained using two methods: from the admittance between the free air gravity and the topography, and from the coherence between Bouguer gravity anomalies and the topography. The first, but not the second, gives values of $T_e$ that are everywhere less than the seismogenic thickness. Where there is sufficient topography, estimates of $T_e$ from PreCambrian shields are all greater than 10 km and do not correlate with the lithospheric thickness. They are probably governed by variations in crustal heat generation rates. Values for regions strongly affected by Phanerozoic tectonics are all less than 7 km, and all such regions are underlain by thin lithosphere.

keywords GOCE gravity, elastic thickness, continental rheology

1. Introduction

The long term elastic behaviour of the lithosphere is generally described by an effective elastic thickness, $T_e$, of an equivalent elastic plate. The value of $T_e$ can be estimated using the relationship between gravity and topography in the spectral domain (e.g. McKenzie and Fairhead 1997). In oceanic regions such values range from $\sim 3$ km for actively spreading ridges to more than 20 km for old ocean lithosphere (Watts 2001). Furthermore in the oceans the seismogenic thickness $T_s$ is everywhere greater than $T_e$ (Watts 2001, Jackson et al. 2008). Corresponding estimates of $T_e$ in

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continental regions are more controversial, and it is with these that this paper is principally concerned. The subject has recently been reviewed by Kirby (2014). The availability of satellite-only gravity fields determined from GOCE (Gravity field and steady state Ocean Circulation Explorer) and other data allows $T_e$ to be estimated everywhere on the continents where there is sufficient topography. Forsyth (1985) argued that the value of $T_e$ could best be obtained from the coherence between Bouger gravity anomalies and topography. Many authors have used his approach, which often gives estimates of 100 km or more in old regions of the continents where there is little topography (Zuber et al. 1989, Bechtel et al. 1990, Wang and Mareschal 1999, Pérez-Gussinyé and Watts 2005, Pérez-Gussinyé et al. 2007).

McKenzie and Fairhead (1997) and McKenzie (2003) argued that $T_e$ can only be estimated using the relationship between gravity and topography in the spectral domain when there is clear coherence between the free air gravity and topography. In many shield regions the two are incoherent at all wavelengths, because most of the topography of shields has been removed by erosion. They argued that it is then not possible to use gravity and topography to estimate $T_e$.

However, there are a few regions of old shields that have substantial topography which is coherent with free air gravity anomalies. This paper analyses the data from several such regions: Africa, Antarctica, the Indian Peninsula, the Siberian Shield, and the Ordos region of NE China, and also from a number of other regions that have been strongly deformed in the Phanerozoic. Though surface gravity measurements are available from some of these regions, in several of them the values are sparse and of uncertain provenance. However, measurements of the gravity gradient tensor by GOCE have now been used to determine the gravity potential coefficients to degree and order (d/o) 300. The resolution of this new gravity model is considerably better than that used by McKenzie et al. (2014) because it incorporates data from the end of the GOCE mission when the orbital height was reduced to about 224 km (Bruinsma et al. 2014). The new gravity model DIR-R5 now allows $T_e$ to be estimated in all continental regions where there is sufficient topography. An additional problem in Antarctica is the ice cover, which conceals the bedrock topography that gives rise to the relevant gravity anomalies. Fortunately there are now sufficient radar measurements of ice thickness to provide a reasonably accurate map of basement topography (Fretwell et al. 2013), which Hirt (2014) has shown correlates with the GOCE gravity field.

In this paper $T_e$ is estimated by two methods from the topography and gravity in the spectral domain using identical boxes, to allow the values from each method to be compared. The first depends on the transfer function, often called the admittance, between the topography and the free air gravity anomalies, using the expressions for the flexure of an elastic sheet subject to surface and internal loads (e.g. McKenzie 2003). The second uses Forsyth’s (1985) approach, and depends on the coherence between the Bouger gravity anomaly and the topography. At least in part, the values of $T_e$ are likely to be controlled by crustal and lithospheric temperatures, both of which are affected by the lithospheric thickness. Therefore the boxes used to estimate $T_e$ are shown superimposed on maps of lithospheric thickness (Priestley and McKenzie 2013), either in the main paper or in the Supplementary Material. This paper is principally concerned with estimates of $T_e$ obtained from two dimensional Fourier transforms of gravity and topography, rather than with one dimensional estimates from profiles. In East Antarctica in particular such estimates depend entirely on GOCE gravity observations.
The standard admittance approach assumes that the Fourier transform of the observed free air gravity anomaly $\tilde{g}_f$ is related to that of the topography $\tilde{t}$ by

$$\tilde{g}_f^n = Z^n(k)\tilde{t} + \pi$$  \hspace{1cm} (1)

where $k = (k_x^2 + k_y^2)^{1/2}$ is the wavenumber, $Z^n$ is the transfer function, which is often called the admittance, and $\pi$ is that part of $\tilde{g}_f^n$ which is incoherent with the topography. $Z$ can then be obtained from

$$Z^n(k) = Z_r^n + iZ_i^n = |Z^n(k)| \exp(i\phi^n) = \frac{\langle \tilde{g}_f^n \tilde{t} \rangle}{\langle \tilde{t}^2 \rangle}$$  \hspace{1cm} (2)

where $\ast$ denotes complex conjugation, and the angle brackets averages over a semicircular annulus $k + \Delta k, k - \Delta k$ in the 2D spectral domain. In general $Z^n$ is complex with real and imaginary parts $Z_r^n$ and $Z_i^n$. The coherency $\Gamma^n$, with real and imaginary parts $\Gamma_r^n$ and $\Gamma_i^n$, is defined as

$$\Gamma^n = \Gamma_r^n + i\Gamma_i^n = \frac{\langle \tilde{g}_f^n \tilde{t} \rangle}{\langle \tilde{t}^2 \rangle} = \frac{1}{2} \left( \frac{\langle \tilde{g}_f^n \tilde{g}_f^n \rangle}{\langle \tilde{g}_f^n \tilde{t} \rangle} \right)^{1/2}$$  \hspace{1cm} (3)

Equations (2) and (3) show that the phase $\phi^n$ of $Z^n$ and $\Gamma^n$ is the same. The (real) coherence $\gamma^2_f$ between the 2D Fourier transforms of the free air gravity and topography in the spectral domain is

$$\gamma^2_f(k) = \frac{\langle |\tilde{g}_f^n \tilde{t} |^2 \rangle}{\langle \tilde{g}_f^n \tilde{g}_f^n \rangle} = \langle \tilde{g}_f^n \tilde{t} \rangle \langle \tilde{g}_f^n \tilde{g}_f^n \rangle$$  \hspace{1cm} (4)

In oceanic regions the value of $T_e$ is generally estimated from the admittance. In contrast, in continental regions almost authors have used Forsyth’s (1985) approach, which depends on the Bouguer coherence, $\gamma^2_b$, calculated from equation (3) using $\tilde{g}_b^n$ instead of $\tilde{g}_f^n$. However, Kirby and Swain (2009) and Audet (2014) have produced maps of $T_e$ using both methods using EGM2008 (Pavlis et al. 2012). Whichever method is used, the values of $\tilde{g}$ and $\tilde{t}$ are required. Two methods have been widely used: Fourier transforms obtained using the Fast Fourier Transform algorithm, generally combined with multitapering (M^cKenzie and Fairhead 1997, Pérez-Gussinyé and Watts 2005), and wavelet methods, using either the derivatives of Gaussian wavelets (Stark et al. 2003) or Morlet wavelets (Kirby and Swain 2008, Audet 2014). Both methods can be used to map spatial variations of $T_e$. However Crosby (2007) showed that spectral estimates from large regions are required to obtain accurate estimates of $T_e$ using the multitaper method when subsurface loads are present, even though it uses windows optimally designed to minimise spectral leakage.

Another important problem is that many shield regions have subdued topography. In such regions there is often no significant coherence between the free air gravity anomalies and the topography. M^cKenzie and Fairhead (1997) and M^cKenzie (2003) argued that it is then not possible to estimate $T_e$ from either the free air or the Bouguer gravity. This result is easily demonstrated. Equation (3) shows that $\gamma^2_f \simeq 0$ requires

$$\langle \tilde{g}_f^n \tilde{t} \rangle = 0$$  \hspace{1cm} (5)

The Bouguer gravity field $g_b^n$ can be constructed from $g_f^n$ and $t$

$$g_b^n = g_f^n - At$$  \hspace{1cm} (6)
where $A$ is a constant. If there is no coherence between the free air gravity and topography, then equations (4), (5) and (6) require

$$
\gamma_2^2(k) = \frac{1}{1 + R(k)}
$$

where

$$
R(k) = \frac{\langle \mathcal{F}g_0^2 \mathcal{F}\rangle}{\langle At^* A\mathcal{T}^* \rangle}
$$

is the ratio of the power in the observed free air gravity field, $g_0^2$, to that of the gravity field from the surface topography, $At$, in the relevant wavenumber band. The Fourier transform of the free air gravity anomaly $\mathcal{F}g_0$ would be given by $At$ if the surface topography was the only load and there was no compensation. Since $t$ is incoherent with $g_0^2$, further erosion will decrease $t$ but will scarcely affect $g_0^2$. Hence $R(k)$ will increase, as will the estimated value of $T_e$. This behaviour accounts for the large values of $T_e$ that many authors have obtained for shields with little topography.

Kirby and Swain (2009) and Audet (2014) identified regions where estimates of $T_e$ were unlikely to be valid using what at first appears to be a different criterion: large values of $\Gamma_0^2/\sqrt{\Gamma_r^2 + \Gamma_i^2}$ or, equivalently, the value of $\phi^o$. Audet masked out such regions in his maps. As Kirby and Swain (2009) point out, there is no physical reason why $Z^o$ should be real and $\phi^o = 0$. However, when the errors $\Delta\phi^o$ are taken into account, none of the estimates of $\phi^o$ for the regions discussed below are significantly different from 0 (see plots of $\phi^o$ in the Supplementary Material). Therefore the observed value of $\phi^o$ is governed by its error $\Delta\phi^o$. When $\gamma_f^2$ is small, $\Delta\phi^o$ is approximately proportional to $1/\sqrt{\gamma_f^2}$ (Munk and Carwright 1966), and therefore becomes large when the coherence is small. This discussion shows that Kirby and Swain’s (2009) reliability condition is essentially the same as that used by M’Kenzie (2003) and M’Kenzie and Fairhead (1997). At present there is no method of estimating $T_e$ from the relationship between gravity and topography when $\gamma_f^2 \approx 0$ throughout the relevant wavelength range.

2. GOCE design and data analysis

The use of gravity models based on data from GOCE to estimate $T_e$ needs to take account of both the design of the spacecraft and of the methods used to derive the spherical harmonic coefficients of the gravity potential from the relevant observations (Rummel et al. 2011, Floberghagen et al., 2011). The principal purpose of the GOCE mission was to obtain an accurate gravity field at wavelengths shorter than that available from GRACE (Gravity Recovery and Climate Experiment). For this purpose the orbit had to be as low as possible. The resulting atmospheric drag was measured using the accelerometers on GOCE and counteracted by an ion thruster that used xenon as a fuel. Since the power for the thruster was supplied by electricity from solar panels, the spacecraft had to be in a face-on orbit that always faced towards the Sun. If the orbit is to retain this geometry throughout the terrestrial year, it must precess once a year as the Earth goes round the Sun. The Earth’s oblateness causes such precession, but only if the satellite orbit is not exactly polar. Therefore GOCE’s orbit was arranged to have an inclination of 96.7°, leaving two polar caps with radii 6.7° uncovered. The average magnitude of a gravity potential coefficient of degree $l$ is approximately
Since the gravity gradient is the second derivative of the potential, its spectrum is approximately white. Short wavelength gravity anomalies are therefore more accurately determined from gravity gradient measurements than they are from orbital perturbations (Rummel et al. 2011). Unlike previous spacecraft, GOCE measured four elements, $V_{xx}$, $V_{yy}$, $V_{zz}$, and $V_{xz}$, of the gravity gradient tensor, where $x$ is in the flight direction, $y$ is horizontal and perpendicular to the orbit plane, and $z$ is radially downwards toward the Earth. These elements of the tensor were measured using three pairs of sensitive accelerometers with baselines of 0.5 m., and all those elements that were well determined were used to obtain the spherical harmonic coefficients of the gravitational potential.

An important source of noise in the GOCE measurements results from orbital perturbations and has frequencies that are harmonics of the orbital period of 5,000 s (Pail et al. 2011). In order to exclude such noise, the GOCE data was filtered with a band pass filter, and only data within the measurement band, of 5 to 100 mHz, was used in the analysis. All data collected from Sept. 1, 2009 to Oct. 20, 2013 was included. That collected after August, 2012, when the orbit was lowered in four steps from its original mean orbit altitude of 254.9 km to 246 km, 240 km, 235 km and finally 224 km, was especially important in determining the short wavelength part of the gravity field (Bruinsma et al. 2014). To obtain the most accurate gravity field from satellite data alone, the GOCE data was combined with data from GRACE and LAGEOS (Bruinsma et al., 2014). Since the boxes used to estimate $T_e$ for Antarctica use data from inside the southern cap south of 83°S, where there is no GOCE data, an important concern is whether the method used to deal with this gap affects the estimated value of $T_e$ for Antarctica. Bruinsma et al. (2014) used the method proposed by Metzler and Pail (2005) to produce DIR-R5, which is complete to degree and order 300.

An alternative method was used to generate GOGRA04S, a satellite-only model that incorporates the same data as that used by Bruinsma et al. (2014). The model consists of a set of spherical harmonic coefficients complete up to degree and order 230. The method of computation was the same as that employed for its predecessor models and is described by Yi (2012) and Yi et al. (2013). Instead of using Metzler and Pail’s approach to fill the polar gaps, a spherical grid with geoid heights was added at both poles above latitude 83°N and below latitude 83°S. These heights were computed from ITG-GRACE2010S (Mayer-Gürr et al. 2010) up to d/o 180. In order to minimize interference of these data with the measured GOCE gradients, the error standard deviation of the geoid grid values was given the rather high value of 20 cm. Estimates of $T_e$ for the two Antarctic boxes were obtained from both DIR-R5 and GOGRA04S, to test whether the method used to deal with the polar gaps affected the resulting values.

The important advantage of using a satellite-only gravity field, rather than one like EGM2008 (Pavlis et al. 2012) that also includes surface measurements, often of uncertain provenance, is that their resolution is uniform and depends only on latitude. Rummel et al. (2011) show a comparison of a satellite-only model that includes GOCE data with EGM2008. There are substantial differences between the two models in central and north Africa, central Asia and South America. The gravity field over the oceans obtained from altimetry provides a useful test of the accuracy of satellite-only gravity models. Fig. 1 shows a comparison of three satellite-only gravity fields with the surface gravity field obtained from altimetry (Smith and Sandwell 1997, Sandwell and Smith 2009, Sandwell et al. 2013) in a region round Hawaii (see McKenzie et al. 2014). To a good approximation the surface of
the Earth is an oblate spheroid with a difference in radius of about 20 km between the polar, $b$, and equatorial, $a$, radius. A height variation of 20 km changes the amplitude of a gravity signal whose wavelength is 200 km by about a factor of two. The polar flattening, of $f = 1 - b/a = 1/298.25$ must therefore be taken into account in the calculation of the surface gravity field from the spherical harmonic coefficients using the expressions given by Moritz (1989). The comparison between the surface gravity and that calculated from DIR-R5 is carried out in the spectral domain, and uses the gravity field, rather than the geoid, in order to emphasise the shorter wavelength signal on which estimates of $T_e$ depend. The improvement in the satellite-only gravity field resulting from the GOCE mission is striking. The relationship between wavelength $\lambda$ and harmonic degree $l$ is approximately $\lambda = 2\pi a/(l+1/2)$, where $a$ is the radius of the Earth. A value of $\lambda$ of 200 km therefore corresponds to $l = 200$. Fig. 1 shows that the transfer function between surface gravity as input and GOCE gravity as output decreases from 0.96 at $\lambda = 184$ km ($l \simeq 217$) to 0.83 at $\lambda = 167$ km ($l \simeq 240$), where the coherence is still as large as 0.97. The coherence only decreases at even shorter wavelengths, becoming 0.83 at $\lambda = 153$ km ($l \simeq 260$). It is therefore probably worth while extending DIR-R5 to produce a model with degree and order greater than 300. Comparisons were also carried out between the gravity field from DIR-R5 and surface measurements from Siberia, the Ordos Plateau, the western US and E. Australia, with results similar to those shown in Fig. 1.

The values of $T_e$ were estimated from the admittance between the free air gravity and topography in the spectral domain, using the approach described by M\textsuperscript{c}Kenzie (2003), with a two layer crust. The projections used are given in the figure captions. The thickness of the upper crust was taken to have a thickness of 15 km and a density of 2700 kg m\textsuperscript{−3}, the lower crust to be 20 km thick with a density of 2900 kg m\textsuperscript{−3}, and the density of the mantle below was assumed to be 3300 kg m\textsuperscript{−3}. Estimates of $T_e$ from the Bouguer coherence used the expressions given by Forsyth (1985).

3. Africa

Fig. 2a shows a map of the lithospheric thickness beneath Africa from PM\textsubscript{v2}_2012 (Priestley and M\textsuperscript{c}Kenzie 2013), together with the three boxes used to estimate $T_e$ in Fig. 3. The topography at wavelengths shorter than $\sim 500$ km in the northern box is dominated by volcanoes, most of which have recently been active. The fit to the flexural model, shown in Fig. 3a, is good, and gives a value of $T_e$ of 23 km. The volcanic loads in this region have been emplaced on a crust that last underwent a major episode of deformation in the Pan African event (650-550 Ma) and much of the region is underlain by lithosphere whose thickness is no greater than $\sim 100$ km (Fig. 2a).

Fig. 3c shows a similar plot for the middle box in Fig. 2a, where the lithospheric thickness is $\sim 150$ km. The estimated value of $T_e$ is 33 km, but the calculated values of admittance are lower than those observed between 400 and 200 km. The value of the admittance in this wavelength band is controlled by the density of the topography, and does not depend on $T_e$. The observed values of $Z$ can be fit by increasing the topographic density to $\sim 3300$ kg m\textsuperscript{−3}. Such a value is greater than that of any surface rocks, and so cannot be the cause of the observed behaviour. What instead is the true explanation is suggested by the admittance values shown by solid dots in Fig. 3c. These were calculated by taking account of the water in Lakes Tanganyika and Malawi, by converting the water into an equivalent layer of rock with a density of 2670 kg m\textsuperscript{−3}. The resulting values of the admittance
are lowered, because the amplitude of the topographic variations is increased. The topography in this part of Africa largely results from rifting. Many of the active and inactive rifts are partly filled by kilometre thicknesses of low density sediments. It is their contribution to the gravity field that generates large negative Bouguer anomalies over African Rift valleys (Bullard 1936, Karner et al. 2000). Because the gravity anomalies from these subsurface density contrasts are coherent with the surface topography generated by rifting, they increase the value of the admittance. It is this effect that leads to the large value of the topographic density. Sadly neither the thickness nor the density of the sediment is known, so a correction cannot be applied. Such subsurface loads are not taken into account by Forsyth’s (1985) approach because they are coherent with the surface topography. Various estimates of $T_e$ for regions of Africa have previously been made. Those based on Bouguer coherence are discussed below. Karner et al. (2000) modelled the flexure of the region around Lake Albert in the space domain, marked with the blue box in Fig. 2b, and estimated $T_e$ to be 24–30 km. This estimate is much greater than that of 6 km obtained by McKenzie and Fairhead (1997) using surface gravity measurements from inside the region in Fig. 2b marked by the small black box. Karner et al. (2000) therefore argued that the admittance method grossly underestimates the value of $T_e$. The surface data has now been reanalysed using the approach proposed by McKenzie (2003), using a two layer crust and allowing internal, as well as surface, loading. The resulting value of $T_e$ is 7 km, with an internal load fraction of 26%. Fig. 2b suggests that the reason why Karner et al.’s and McKenzie and Fairhead’s estimates of $T_e$ from east Africa are different is that there is a difference in lithospheric thickness between the two regions. Estimates of the value of $T_e$ from Bouguer coherence in Table 1 are in general agreement with those from Stark et al. (2003) from South Africa, and with those from Tessema and Antione (2003) from central and northern Africa.

4. Antarctica

The subglacial topography of Antarctica has been extensively mapped using radar, though there are still major gaps in the coverage (Fretwell et al. 2013). The gravity field is now also well resolved. Because GOCE’s orbit has an orbital inclination of 96.7°, the coverage of Antarctica is excellent north of 83° S.

Figs. 4a and b show the resulting gravity field and the equivalent rock topography, calculated from Bedmap2 (Fretwell et al. 2013) by converting the ice thickness and water depth to an equivalent thickness of rock, using a rock density of 2670 kg m$^{-3}$, an ice density of 917 kg m$^{-3}$ and a sea water density of 1030 kg m$^{-3}$, then adding this rock thickness to the bedrock topography. Fig. 4b shows the resulting topography. An unexpected feature of Fig. 4b is that large parts of E. Antarctica then have an elevation of more than 1 km. The topography is rougher than is that of most other PreCambrian shields. Erosion by moving ice generates completely different landforms to those formed by river erosion (Creyts et al. 2014). Ice erosion is strongly affected by whether liquid water is present at the ice-rock interface. The direction of ice movement is largely controlled by the slope at the ice surface and by the boundary conditions at the ice-rock interface, rather than by the topography of the ice-rock interface. Furthermore erosion by ice often occurs below sea level. Creyts et al. (2014) believe that many of the landforms beneath central Antarctica have undergone much less erosion than they would have done if they had been subaerial.

Whatever the explanation, Fig. 4b shows that the equivalent topography of Antarctica is rough,
and Hirt (2014) first showed that it is coherent with the GOCE gravity field. This behaviour has been used to calculate the admittance for the boxes in E. and W. Antarctica shown in Fig. 4a. Because of the incomplete coverage, the equivalent rock topography is not free from noise, and because part of the gravity field results from subsurface density contrasts that are treated as noise in the gravity field, the admittance $Z$ in both boxes was calculated from

$$Z_2 = \left[ \frac{< \bar{g} \bar{g} f >}{< \bar{t} \bar{t} f >} \right]^{1/2}$$

instead of from the usual expression, equation (2), which assumes that the topography is free from noise. This condition is unlikely to be true for Bedmap2 because of the limited coverage of radar soundings and direct thickness measurements.

The open circles in Figs. 5a and c show the admittances calculated from GOGRA04S, instead of DIR-R5. At wavelengths greater than 200 km the values from the two models are indistinguishable, and therefore the estimated values of $T_e$ are not affected by the method used to fill the hole in the GOCE data at the S. Pole. The estimated value of $T_e$ for E. Antarctica is about 21 km (Fig. 5a). The misfit in Fig. 5b shows that larger values of $T_e$ fit the observations only slightly less well than does a value of 21 km. Therefore the lower, but not the upper, bound on the value of $T_e$ is well constrained. The lithospheric thickness beneath E. Antarctica is also not yet well constrained: Priestley and McKenzie’s values vary from about 140 km near the coast to 200 km on the western margin of the box in Fig. 4a. The value of $T_e$ is therefore slightly smaller than is that for the southern African region in Fig. 2a, and the lithospheric thickness is slightly greater. An important issue is whether the admittance estimates are affected by any ice removal that may have occurred. The values of admittance at long wavelengths in Fig. 5a are 40–50 mGals/km, or in the expected range for convective support. In contrast, topography that has not yet rebounded from ice loading has an admittance $\sim$ 140 mGals/km (McKenzie 2010). It is therefore unlikely that the value of $T_e$ estimated from the admittance in Fig. 5a is strongly affected by such topography. This argument also suggests that gravity and topography with wavelengths greater than $\sim$ 600 km is convectively supported.

Figs. 5a and c show that there is a striking contrast between the estimates of $T_e$ from E. and W. Antarctica. The value for W. Antarctica, of 5 km, also estimated from the value of $Z_2$, is well constrained by the observations. Its low value and the thin lithosphere are both probably the result of past subduction along the Pacific margin.

5. Other Regions

Two estimates of $T_e$ were obtained from the Peninsula of India using DIR-R5 (Fig. 6). The northern region is strongly affected by flexural loading by the Himalaya. McKenzie and Fairhead (1997) obtained a value of 24 km for $T_e$ for this box, in agreement with that of 25 km from DIR-R5. The southern box was chosen to cover approximately the same region where Tiwari and Mishra (1999) collected three E-W gravity traverses across the Peninsula to estimate $T_e$. They used the admittance method, and obtained a value of $10 \pm 2$ km, which agrees with the value of 10 km from DIR-R5 (Fig. 6).

The box underlain by the Siberian Shield is the same as that used by McKenzie and Fairhead (1997, Fig. 7). The value of $T_e$ they obtained, of 16 km from surface data, whose coverage in
this region is good, agrees with that of 14 km from DIR-R5. Reanalysis of the surface data using M'Kenzie's (2003) methods gives $T_e = 12$ km. The boxes used for E. Australia and the western US are similar to those used by M'Kenzie and Fairhead. Maps of all three are shown in the Supplementary Material. That used for the N. China Craton is illustrated in Fig. 7. In general Table 1 shows that the values of $T_e$ obtained from GOCE and surface gravity agree well. They also agree with those obtained using the admittance and Forsyth's method by other authors. However, no regions where the free air gravity is incoherent with the topography have been included in the Table.

6. Discussion

The estimates of $T_e$ obtained from the admittance and from the Bouguer coherence for the various regions discussed above are listed in Table 1. Also listed is the depth of the deepest earthquake inside or close to the relevant box and the average lithospheric thickness. The values of $T_e$ given in this table, and the plot of the lithospheric thickness as a function of $T_e$, raise two important issues. The first is which value of $T_e$ is more accurate: that from admittance or that from Bouguer coherence? The second issue is why there is so little correlation between $T_e$ and lithospheric thickness for the PreCambrian Shields in Fig. 8b, in contrast to the Phanerozoic regions in Fig. 8a, which have both thin lithosphere and small values of $T_e$.

Estimates of $T_e$ in Table 1 obtained using Bouguer coherence are about twice those obtained from the admittance. The same is true of those obtained from surface data by M'Kenzie and Fairhead (1997). However, Kirby and Swain (2008) did not find this behaviour when they analysed synthetic data sets. The principal reason why we believe those from the admittance method are likely to be the more accurate than that from the Bouguer coherence concerns the ratio of $T_e$ to $T_s$, the seismogenic thickness. The stress released by earthquakes needs only to be stored elastically for times of $\sim 10^3$ a, whereas that involved in topographic support involves times of $10^7 - 10^9$ a. Therefore $T_e$ should be smaller than $T_s$, in agreement with the results from oceanic regions, where the ratio $T_e/T_s \sim 0.5 - 0.7$ (Jackson et al. 2008, Watts 2001). The same is true of continental regions if the value of $T_e$ is determined from the admittance, but, as Table 1 shows, not if $T_e$ is obtained from Bouguer coherence, when in many cases $T_e > T_s$. Burov (2010) disagrees strongly with this argument, and sees no rheological reason to believe that $T_e < T_s$. His view depends on a number of assumptions, and does not explain why this condition should be satisfied in oceanic, but not in continental, regions. A simpler explanation is that Forsyth’s method overestimates $T_e$, even when there is appreciable coherence between the free air gravity and the topography. If this is indeed the case, and the admittance method provides valid estimates of $T_e$, then $T_e < T_s$ for both continents and oceans.

The results discussed above for the central African box show that a correlation between surface and subsurface loads can result from sedimentation, as well as from erosion. The modelling that Karner et al. (2000) carried out in the space domain shows that the Albertine Rift Valley is underlain by several kilometres of low density sediment. Such sediments form a subsurface load that correlates with the surface topography, contradicting Forsyth’s (1985) assumption. Loads of this type cause the values of the admittance to exceed those expected from the density of surface rocks (Fig. 3c), and are likely to be common in areas, like the East African Rift, that are presently undergoing extension. However, such loads only have a limited effect on the estimated value of $T_e$. 

9
The lack of correlation between the lithospheric thickness and $T_e$ in PreCambrian regions, Fig. 8b, but not for Phanerozoic ones in Fig. 8a, is at first rather surprising. It probably results from crustal temperatures being more strongly affected by variations in crustal heat production than by those of lithospheric thickness where the lithosphere is thick. The heat generation rates within the upper and lower crust are 1.70 and 0.26 $\mu$W m$^{-3}$ respectively, calculated from Rudnick and Gao’s (2005) estimates of their average composition, and Jaupart and Mareschal’s (2005) expressions for the heat production from K, U and Th. Figs. 8c and d show geotherms for a mantle potential temperature of 1315°C for various lithospheric thicknesses and crustal heat generation rates. The curves in Fig. 8d show the effect of using upper and lower crustal heat generation rates throughout the whole crust. The resulting range of temperatures at depths of 25 and 35 km are 260 – 580°C and 346 – 693°C. These ranges are greater than those in Fig. 8c, which show that lithospheric thicknesses of 99, 126 and 199 km, corresponding to the range in Table 1, produce temperature variations of 491, 401, and 325°C at 25 km and 623, 501, and 394°C at 35 km. Jaupart and Mareschal (2005) show that the surface heat flux of shields is very variable, as a result of variations in crustal heat production. The two crustal models shown in Fig. 8d have surface heat fluxes of 29 and 79 mW m$^{-2}$, of which only 19 and 11 mW m$^{-2}$ come through the Moho. The corresponding values for the geotherms in Fig. 8c, with lithospheric thicknesses of 99, 126 and 199 km, are 70, 60 and 52 mW m$^{-2}$ at the surface, and 30, 23 and 15 mW m$^{-2}$ at the Moho. These results show that the values of $T_e$ are likely to be more sensitive to variations in crustal heat production than to those of lithospheric thickness when it is 150–200 km. In contrast, the estimates of $T_e$ from regions that have been strongly affected by Phanerozoic tectonics are all less than 7 km. Because of its limited vertical resolution, surface wave tomography cannot yet determine the lithospheric thickness in continental regions when it is less than ~ 100 km. The values of lithospheric thickness for such regions listed in Table 1 are therefore upper bounds. Fig. 8c shows that the crustal temperatures in these regions are probably governed more by lithospheric thickness than by variations in crustal heat generation.

There is as yet no obvious method of using values of $T_e$ to estimate the temperature $\Theta_e$ at which elastic stresses cease to be maintained for times of $10^7 – 10^9$ years. Jackson et al. (2008) argued that the base of the seismogenic layer corresponds to a temperature $\Theta_s$ of 350 ± 100°C in regions affected by Phanerozoic tectonics, and that it must be greater beneath PreCambrian shields. If $T_e < T_s$, as the admittance modelling suggests, then $\Theta_e$ is likely to be less than $\Theta_s$. The same conclusion follows from rheological considerations, since stresses that generate earthquakes only have to be maintained for $10^3 – 10^5$ years. Sadly none of the methods that have been used to estimate $T_e$ can determine the depth to the relevant elastic layer.

7. Conclusions

Satellite-only gravity models, derived using data from LAGEOS, GRACE and GOCE satellites, now have sufficient resolution to allow $T_e$ to be estimated for all continental regions where there is sufficient topography. Values for PreCambrian Shields, determined from the admittance between the topography and free air gravity, are between 10 and 33 km, and 25–50 km using Bouguer coherence. Corresponding values from regions have been strongly affected by Phanerozoic tectonics are 4 – 7 km from admittance and 12 – 25 km from Bouguer coherence. Regions where there is sufficient surface
gravity, such as Siberia, the western USA and eastern Australia, the estimates of $T_e$ from surface and satellite data agree (Table 1). In oceanic regions $T_e$ is usually obtained from estimates of the admittance and, as expected, is less than the seismogenic thickness $T_s$. The same is true for continental regions when $T_e$ is estimated from the admittance, but not when the Bouguer coherence method is used. Partly for this reason, and partly because some of the assumptions involved in the Bouguer coherence method are unlikely to be satisfied, estimates of $T_e$ obtained using the admittance method are probably more accurate. Beneath Precambrian shields the value of $T_e$ does not correlate with lithospheric thickness, probably because it is principally controlled by variations in rates of crustal heat production. However, beneath regions strongly affected by Phanerozoic tectonics $T_e \leq 7$ km, probably because crustal temperatures in such regions are principally controlled by the thickness of the lithosphere.

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**PreCambrian**

|   |   |   |   |   |
|---|---|---|---|
| 12 | E. Africa | S | 6.6 | 5.6–7.2 | 12 | 28 | 102 ± 8 |
| 13 | W. Antarctica | G | 5.4 | 5.0–5.4 | 12 | – | 99 ± 6 |
| 14 | W USA | G | 4.6 | 4.2–5.0 | 16 | 31 | 116 ± 28 |
| 15 | W USA | S | 4.4 | 3.8–4.6 | 16 | 31 | 116 ± 28 |
|   | Audet (2014) W. USA | S | <20 | – | <20 |   |   |
|   | Kirby & Swain (2009) | S | <10 | – | <10 |   |   |
| 16 | Eastern Australia | G | 6.5 | 0.8–5.5 | 25 | 18 | 101 ± 14 |
| 17 | Eastern Australia | S | 6.5 | 4.5–8.5 | 25 | 18 | 101 ± 14 |
|   | Audet (2014) E. Australia | S | <20 | – | 20-30 |   |   |
|   | Zuber et al. (1989) | S | – | – | 16-64 |   |   |
| 18 | Hawaii | S | 18.2 | 16.8–19.4 | 53 | 100 |   |
All spectra were obtained using the multitaper method, with three tapers in the $x$ and $y$ directions, and a frequency bandwidth of 4. The best fitting value of $T_e$ was calculated by finding the minimum misfit of $H$, $H_{\text{min}}$, where

$$H = \left[ \frac{1}{N} \sum_{i=1}^{N} \left( \frac{Z_i^o - Z_i^c(T_e)}{\sigma_i} \right)^2 \right]^{1/2} \quad (10)$$

$Z_i^o$ is the observed value of the admittance with standard deviation $\sigma_i$ and $Z_i^c$ is the calculated value. The range of values of $T_e$ in column 5 was obtained by finding the values of $T_e$ on each side of the minimum of $H$ that gave values of $H = 1.2H_{\text{min}}$. The values of $T_e$ listed in the column labelled Boug(uer) coher(ence) were obtained using Forsyth’s (1985) expressions.

The seismogenic thickness $T_s$ was obtained from the depth of the deepest earthquakes in the relevant region. In Africa, Siberia and India, depths constrained by waveform modelling are available (Craig et al. 2011, Sloan et al. 2011, Jackson personal communication 2014). In the other regions the depths listed in the catalogue of Engdahl et al. (1998 and later additions) were used. Engdahl et al. (2006) compared the depths from the catalogue with those from waveform modelling, and showed that, if the catalogue depths were restricted to those listed with the code ‘DEQ’, they were within ±10 km of those from waveform modelling. In N Africa, S. India and Antarctica there are no earthquakes with well constrained depths.
Figure 1: Comparison of three satellite-only gravity models with surface gravity from altimetry for the region surrounding Hawaii (McKenzie et al. 2014). (a)-(c) show the value of the admittance, calculated from the surface gravity as input and the gravity calculated from the satellite gravity model as output, and (d) shows the coherence between DIR-R5 and the surface gravity.
Figure 2:
Figure 2: (a) Contours of the lithospheric thickness (Priestley and McKenzie 2013) for Africa, showing the boundaries of the three boxes used to estimate the values of $T_e$ in Fig. 3. The black dots show the locations of the epicentres of earthquakes whose depth, determined from wave form modelling, is 20 km or more (Craig et al. 2011). Airy projection, centred on $0^\circ$N, $15^\circ$E, $\beta = 35^\circ$. (b) As for (a), but also showing in blue the box used by Karner et al. (2000) to estimate $T_e$ in the space domain, and, in black, that used by McKenzie and Fairhead (1997) for the same purpose in the spectral domain. Mercator projection.
Figure 3: Admittance and misfits for the N. African, (a) and (b), Central and southern African, (c) and (d), and South African, (e) and (f) boxes in Fig. 2a. The open circles in (c) show the admittance when the topography is taken as the surface of Lakes Tanganyika and Malawi, and the filled circles when the water in these lakes is converted into an equivalent thickness of rock with a density of 2670 kg m\(^{-3}\). The wavelength band used to calculate the misfit was 200-640 km.
Figure 4:
Figure 4: (a) Lithospheric thickness (see Fig. 2 for details) for Antarctica, showing the boxes used to estimate $T_e$ for E. Antarctica (right box) and W. Antarctica (left box). The black circle shows the region where there are no GOCE data. (b) Equivalent topography of Antarctica, produced by converting the ice and sea water thicknesses into rock using an ice density of 917 kg m$^{-3}$, a water density of 1030 kg m$^{-3}$, and a rock density of 2670 kg m$^{-3}$. Lambert equal area projection centred on $-90^\circ$N.
Figure 5: Admittance and misfits calculated from DIR-R5 (solid circles) and GOGRA04S (open circles) for E. and W. Antarctica, using equ. (9). The wavelength band used to calculate the misfit was 230-430 km.
Figure 6: As for Fig. 2, showing two boxes used to estimate $T_e$ for the Indian Peninsula. The black dots show earthquake epicentres whose depth, determined from waveform modelling, is 20 km or more (Jackson, personal communication). Airy projection centred on 20°N, 77°E with $\beta = 15^\circ$. 

25
Figure 7: As for Fig. 2. The epicentres of earthquakes with depths of 20 km or more are from Engdahl et al. (1998) and updates. Airy projection centred on 33°N, 112°E with $\beta = 20^\circ$. 
Figure 8: (a) and (b) average lithospheric thickness and its standard deviation, as a function of $T_e$, for the regions listed in Table 1. The open circles show estimates from surface gravity data, the solid ones from DIR-R5. The lower bound on the lithospheric thickness in (a), shown by the arrows, is unconstrained by surface wave tomography. Notice that the horizontal scale in (a) differs from that in (b). (c) Geotherms for three lithospheric thicknesses. In all cases the crust consists of two layers, each 20 km thick, with heat generation rates of $1.70 \mu W m^{-3}$ for the upper layer and $0.26 \mu W m^{-3}$ for the lower. (d) Geotherms for two values of crustal radioactive heat generation. In both cases the crustal thickness is 40 km and that of the lithosphere is 176 km. The heavy dashed horizontal line in (c) and (d) shows the cutoff temperature for earthquakes in the oceanic mantle (Jackson et al. 2008).