

Supporting Information for “CO₂ Dissolution Trapping Rates in Heterogeneous Porous Media”

K. A. Gilmore¹, J. A. Neufeld^{1,2,3}, M. J. Bickle⁴

¹Bullard Laboratories, Department of Earth Sciences, University of Cambridge, Madingley Rise, Madingley Road, Cambridge, CB3

0EZ, UK

²BP Institute, University of Cambridge, Madingley Road, Cambridge, CB3 0EZ, UK

³Institute of Theoretical Geophysics, Department of Applied Mathematics and Theoretical Physics, University of Cambridge,

Cambridge, CB3 0WA, UK

⁴Department of Earth Sciences, University of Cambridge, Downing Street, Cambridge, CB2 3EQ, UK

1. Scalings of Total Dissolution with Time

The length of the finger is governed by the input flux and the amount of dissolution. Figure S1b shows how the length of the CO₂ finger and the total lateral dissolution of CO₂ scale with time. At early times ($t \ll 1$), when the dissolution area to input flux ratio of the CO₂ finger is small, the amount of dissolution is negligible and so the length evolves as $L \sim t$. The diffusive CO₂ profile away from the CO₂-water interface (in the z direction) scales like $t^{-1/2}$, hence the total diffusive flux F_{total} scales as,

$$F_{total}(t) \sim \int_0^{L(t)} F dx \sim t^{1/2}. \quad (1)$$

The total lateral dissolution of CO₂, S_{tot} , into the low permeability layers is the total mass of CO₂ that dissolves over time. This is equal to the total CO₂ flux across the whole high/low permeability interfacial area over time, and the scaling at early times is

$$S_{tot}(t) \sim \int_0^t F_{total} dt \sim t^{3/2}. \quad (2)$$

At late times ($t \gg 1$), the length tends to evolving as $L \sim t^{1/2}$ as diffusive loss dominates. The diffusive flux F at a point x on the finger continues to scale like $F \sim t^{-1/2}$, which means the total diffusive flux tends to a constant. Hence, the total lateral dissolution into the low permeability layers scales as $S_{tot} \sim t$ at late times.

For the multiple finger case, at early times ($t \ll 1$) the scaling for the total dissolution is the same as the single finger case, and goes like $S_{tot} \sim t^{3/2}$. At late times, the CO₂ saturation of water in the low permeability layers stops diffusion in the proximal parts of the CO₂ layers, and the system enters a steady state with a constant length zone at the front of the CO₂ finger in which dissolution of CO₂ occurs. The total diffusive flux over this zone is constant, hence the total lateral dissolution into the low permeability layers scales as $S_{tot} \sim t$ at late times (Figure S2b).

Note that the total dissolution plotted in Figure S1 and Figure S2 only includes lateral CO₂ dissolution from the high permeability layer into the surrounding water in the low permeability layers and not dissolution of CO₂ into the residual water remaining in the high permeability layer.

Figure S1. (a) Length of the CO₂ finger L as a function of time t for the single finger case. (b) The total lateral dissolution of CO₂ from the high permeability finger into the surrounding water is plotted as a function of time. At early times, the total dissolution scales with $t^{3/2}$. At late times, the total dissolution tends towards being proportional to time. Both graphs are plotted for $\alpha = 0, 0.01, 0.1$.

Figure S2. (a) Length of the CO₂ fingers L as a function of time t for the multiple finger case. (b) The total lateral dissolution from the CO₂ finger is plotted as a function of time. At early times, the total dissolution scales with $t^{3/2}$. At late times, the total dissolution evolves proportional to time, as the diffusive flux becomes constant. Both graphs are plotted for $\alpha = 0, 0.01, 0.1$ and $\beta = 5$